

$$① \quad \begin{cases} -x'' + p(x) x' + q(x) x = r(t) & (Lx=r) \\ x(a)=\alpha \\ x(b)=\beta \end{cases}$$

$$y_1: \quad \begin{cases} Ly_1 = r \\ y_1(a) = \alpha \\ y_1(b) = 0 \end{cases} \quad (PVI1)$$

$$y_2: \quad \begin{cases} Ly_2 = 0 \\ y_2(a) = 0 \\ y_2(b) = 1 \end{cases} \quad (PVI2)$$

Resoudre les problemes de valeur initial, nous avons $y(t) = u(t) + v(t)$

$$\begin{aligned} u' &= v \\ v' &= pu + qv - r \end{aligned} \quad \begin{pmatrix} u' \\ v' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ q & p \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} + \begin{pmatrix} 0 \\ -r \end{pmatrix}$$

$$Res (PVI1) \quad \begin{pmatrix} u_1 \\ v_1 \end{pmatrix} = \begin{pmatrix} \alpha \\ 0 \end{pmatrix} + (b-a) \begin{pmatrix} 0 \\ \alpha q_0 - r_0 \end{pmatrix}$$

Approximé de $y_1(b) \approx \alpha$
 $y_1(b) \approx (b-a)(\alpha q_0 - r_0)$

$$Res (PVI2) \quad \begin{pmatrix} u_1 \\ v_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} + (b-a) \begin{pmatrix} 1 \\ p_0 \end{pmatrix}$$

Approximé de y_2 $y_2(b) \approx (b-a)$
 $y_2(b) \approx 1 + (b-a)(p_0 - r_0)$

Approximé de $x(b)$
 $x(b) = y_1(b) + \frac{(b-y_1(b))}{y_2(b)} y_2(b)$

$$\begin{aligned} y_1 + Sy_2 &= x \\ y_1(b) + Sy_2(b) &= \beta \\ S &= \frac{(b-y_1(b))}{y_2(b)} \end{aligned}$$

2

$$\begin{cases} -x'' + 3x' + 4x = 4 \\ x'(0) = -e \\ x(0) = 0 \end{cases}$$

$$\begin{cases} -x'' + 3x' + 4x = 0 \\ x'(0) = 0 \\ x(0) = 1 \end{cases}$$

Solution des problèmes

$$x(t) = u(t) + 5v(t) \\ u(t) + 5v(t) = 2 \Rightarrow 5 = \frac{2 - u(t)}{v(t)}$$

↓
solution particulière $u(t)$

↓
solution homogène $v(t)$

$$v(t) = \frac{1}{5}e^{4t} + \frac{2}{5}e^{-t}, \quad v(0) \neq 0$$

① $\exists$ problème homogène trouvé lors de la solution trouvée

Unicité : $\begin{cases} \textcircled{1} \\ \textcircled{2} \end{cases} \quad v(0) \neq 0$

Differences finies

$$\begin{array}{ccccccc} & & 0 & 1 & 1 & 1 & 1 \\ -h & 0 & 1 & 1 & 1 & 1 & 1 \\ & & 0 & 1 & 2 & & \\ & & & & & & 1 \\ & & & & & & 2 \\ & & & & & & \vdots \\ & & & & & & N+1 \end{array}$$

Se introduit $t_1 = -h$

$$\frac{x_1 - x_{-1}}{2h} = -e \Rightarrow -x_{-1} + x_1 = -2he$$

$$\begin{pmatrix} 1 & 0 & -1 & 0 & \dots & \dots \\ -1 + \frac{3h}{2} & 2+4h^2 & -1 + \frac{3h}{2} & 0 & \dots & \dots \\ -1 + \frac{3h}{2} & 2+4h^2 & -1 + \frac{3h}{2} & 0 & \dots & \dots \\ & & & -1 + \frac{3h}{2} & 2+4h^2 & \dots \\ & & & & & \ddots \end{pmatrix} \begin{pmatrix} x_{-1} \\ x_0 \\ x_1 \\ \vdots \\ x_N \end{pmatrix} = \begin{pmatrix} 2he \\ 4h^2 \\ \vdots \\ \vdots \end{pmatrix}$$

$$\begin{pmatrix} 2+4h^2 & -2 & 0 & \dots & \dots \\ -1 + \frac{3h}{2} & 2+4h^2 & -1 + \frac{3h}{2} & \dots & \dots \\ & & & & \ddots \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ \vdots \\ x_N \end{pmatrix} = \begin{pmatrix} 4h^2 + 2he(1 + \frac{3h}{2}) \\ 4h^2 \\ \vdots \end{pmatrix}$$

Equal

$$2+4h^2 > 1 + \frac{3h}{2} + \left| \frac{3h}{2} - 1 \right| = \begin{cases} 3h \\ 2 \end{cases} \text{ soupreorne.}$$

UNIVERSITÉ DE BORDEAUX
FACULTÉ DES SCIENCES
LABORATOIRE DE MATHÉMATIQUES
UMR 5801 - MATHÉMATIQUES DE BORDEAUX

(3)

$$x' = 2t + 3$$

$$x(0) = 4$$

$$\Rightarrow x(t) = t^2 + 3t + 4$$

a)

$$x_n = x_{n-1} + h(2t_n + 3) = x_{n-1} + h(2hn + 3)$$

$$x_n = 4 + [(n+1)h + 3]nh \quad \text{h reurs for induction}$$

$$x_0 = 4 \quad \text{OK}$$

$$\text{S's with } n-1: \quad x_{n-1} = 4 + [nh + 3](n-1)h$$

$$x_n = 4 + [nh + 3](n-1)h + h(2hn + 3) = 4 + h[n(n-1)h + 3n - 3 + 2hn + 3] =$$

$$= 4 + hn[(n-1)h + 3 + 2h] = 4 + [(n+1)h + 3]nh.$$

b)

$$E_n = t_n^2 + 3t_n + 4 - 4 - [(n+1)h + 3]nh = nh(nh + 3 - (n+1)h - 3) = -nh^2$$

$$|E_n| = nh^2 \quad \max |E_n| = nh^2 = h \Rightarrow \boxed{\Sigma(h) = h}$$

(4)

$$a) \alpha_0 + \alpha_1 + \alpha_2 = 0$$

$$\alpha_1 + 2\alpha_2 = 1 + \beta$$

$$\alpha_1 + 4\alpha_2 = 4 + 2\beta$$

$$\alpha_1 + 8\alpha_2 = 12 + 3\beta$$

$$\alpha_1 + 16\alpha_2 = 32 + 4\beta$$

...

$$\begin{pmatrix} 1 & 1 & 1 & | & 0 \\ 0 & 1 & 2 & | & 1+\beta \\ 0 & 1 & 4 & | & 4+2\beta \\ 0 & 1 & 8 & | & 12+3\beta \\ 0 & 1 & 16 & | & 32+4\beta \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & | & 0 \\ 0 & 1 & 2 & | & 1+\beta \\ 0 & 0 & 2 & | & 3+\beta \\ 0 & 0 & 4 & | & 8+\beta \\ 0 & 0 & 8 & | & 20+\beta \end{pmatrix}$$

Los tres últimos son incompatibles $\forall \beta$.

La 3ª y 4ª compatibles $\Rightarrow \beta = 2$, $\alpha_0 = -1/2$, $\alpha_1 = -2$, $\alpha_2 = 5/2$
orden 3

$$b) \beta = -1/2 \quad \Downarrow \text{orden 2} \quad 5z^2 - 8z + 3 = 0 \Rightarrow z = 1 \quad \text{o-estable} \quad \Rightarrow \text{convergente}$$

Región de estabilidad: $(5-4\bar{h})z^2 + (2\bar{h}-8)z + 3 = 0 \quad \bar{h} < 0$

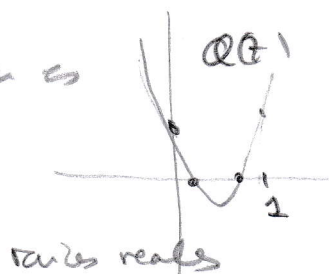
Si $\bar{h} < 0$ $Q(z)$ representa una parábola convexa

$$Q(0) = 3 > 0$$

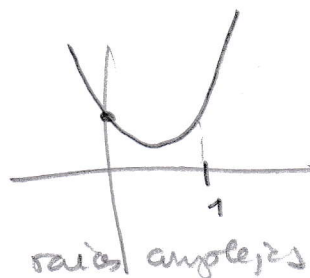
$$Q(1) = -2\bar{h} > 0$$

$$Q'(0) = 2\bar{h} - 8 < 0$$

$$Q'(1) = 2 - 6\bar{h} > 0 \quad \text{La estructura es}$$



o bien



Si las raíces de $Q(z)$ son reales están comprendidas entre 0 y 1

Si son complejas: $s_i = \frac{4-\bar{h} \pm i\sqrt{-(\bar{h}^2+4\bar{h}+1)}}{5-4\bar{h}}$, su módulo es

$$|s_i| = \frac{(4-\bar{h})^2 - (\bar{h}^2 + 4\bar{h} + 1)}{(5-4\bar{h})^2} = \frac{15-12\bar{h}}{(5-4\bar{h})^2}; \quad |s_i| < 1 \Leftrightarrow$$

$$\Leftrightarrow 15-12\bar{h} < 25-40\bar{h}+16\bar{h}^2 \Leftrightarrow 0 < 10-28\bar{h}+16\bar{h}^2 \quad \text{OK}$$

y $\bar{h} < 0$

c) Se trata de un predictor-corrector con el implícito del apartado b) y Euler explícito

orden = 2.